Methods to Measure the Reaction Cross Section

Riccardo Raabe
KU Leuven, Instituut voor Kern- en Stralingsfysica

27 EU PRISMAP Medical Radionuclides

PRISMAP School on radionuclide production
Leuven, 27-31/05/2024

KU LEUVEN

Contents

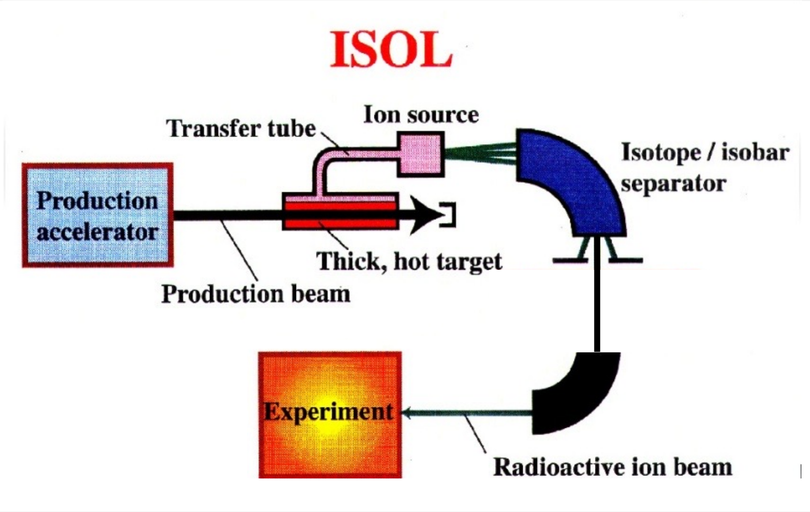
- Production of radioactive ion beams
  and production of radioisotopes
- The cross section
  - Definition
  - Measured quantities
- Nuclear reactions
  - Definitions
  - Probability of different processes
- Production of unstable nuclei
  - Fusion
  - Other processes
- Current production methods
- Yields at facilities
- Summary/conclusions

KU LEUVEN

Production of radioactive ion beams

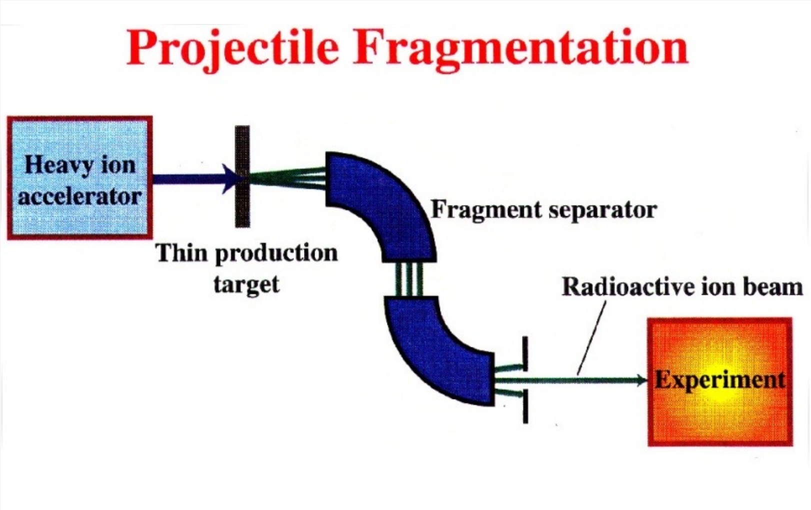
Isotope Separation On-Line

- (Mostly) light ion beam on heavy-ion target
- Products are stopped in the target
- **Depends on chemistry**
- **Slow** (diffusion from the target)



In-flight separation

- Heavy-ion beam on thin target
- **Fast**
- Used to study very short-living isotopes, produced with small probabilities

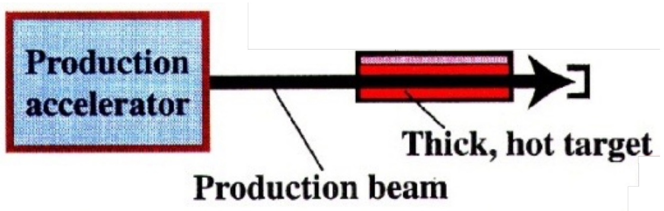


KU LEUVEN

Production of radioisotopes

Half life not so crucial → two separate moments

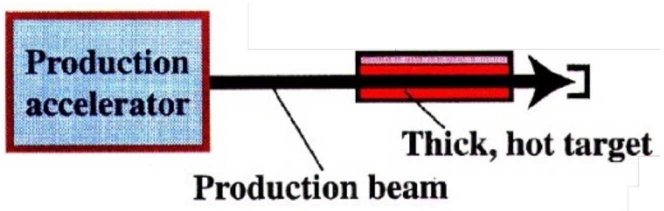
Production in thick target



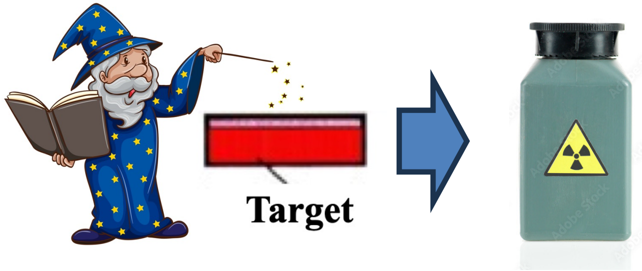
Production of radioisotopes

Half life not so crucial → two separate moments

Production in thick target



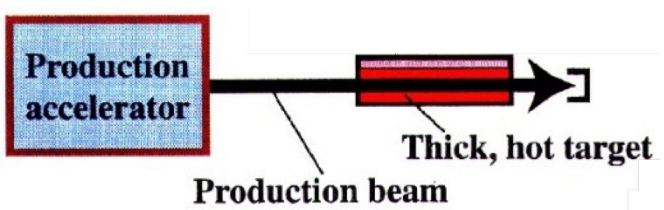
Extraction, separation



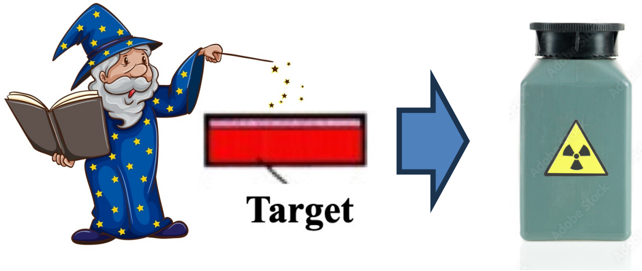
Production of radioisotopes

Half life not so crucial → two separate moments

Production in thick target



Extraction, separation



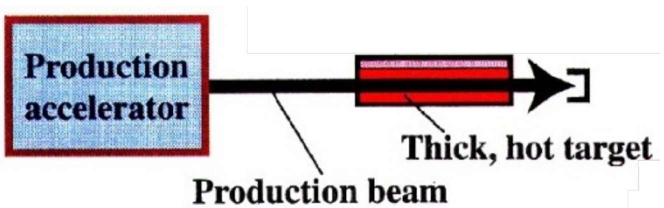
Yield of radioisotopes: $Y = \sigma \, N_t \, \Phi \;\; \epsilon$

- σ: cross section
- N_t: target nuclei
- Φ: primary beam intensity
- ϵ: separation efficiency

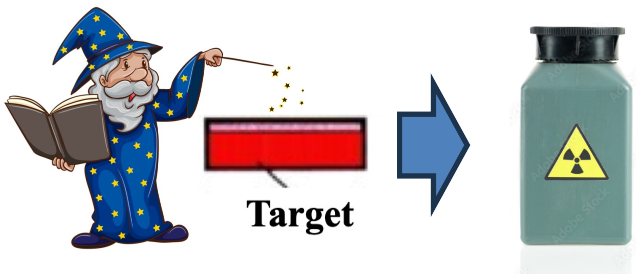
Production of radioisotopes

Half life not so crucial → two separate moments

Production in thick target



Extraction, separation



Yield of radioisotopes: $Y = \sigma N_t \Phi \epsilon$

- σ: cross section
- N_t: target nuclei
- Φ: primary beam intensity
- ϵ: separation efficiency

Cross section

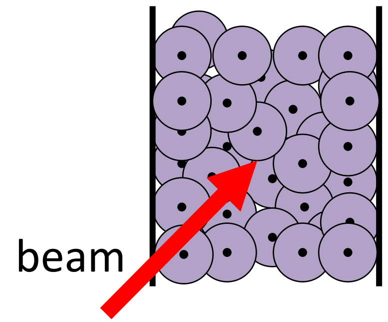
$$Y = \sigma \, N_t \, \Phi \, \epsilon$$

yield: $[T]^{-1}$

cross section: $[L]^{2}$ units: barn ($1 \text{ b} = 10^{-24} \text{ cm}^2$); mb; fm^2

target nuclei: $[L]^{-2}$

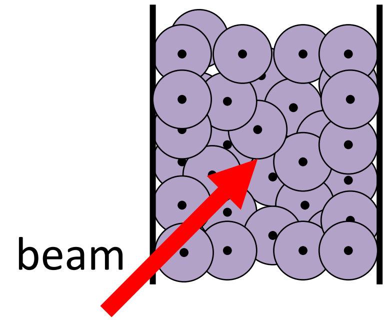
primary beam intensity: $[T]^{-1}$



Cross section

$$Y = \sigma \, N_t \, \Phi \;\; \epsilon$$

- yield: $[T]^{-1}$
- cross section: $[L]^{2}$
- target nuclei: $[L]^{-2}$
- primary beam intensity: $[T]^{-1}$



Quantum mechanics

$|\Psi_{\mathrm{out}}|^2 dV$ (yield)

$|\Psi_{\mathrm{in}}|^2 dV$ (primary beam intensity)

Solve Schrödinger equation by:

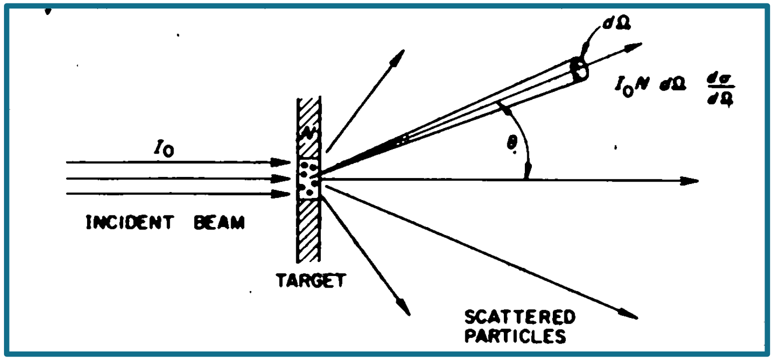
- separation between internal degrees of freedom and relative motion
- boundary conditions for relative motion (conservation principles)
- build ad-hoc potential that describes the interaction between the nuclei

KU LEUVEN

Cross section

We can measure…

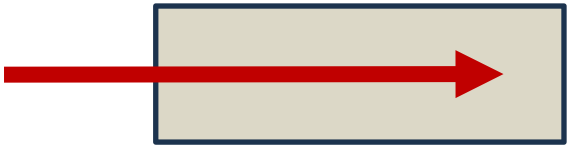
- Cross section as function of the angle:
  angular distribution $d\sigma/d\Omega$



- Total cross section for all possible channels
  → attenuation of the beam intensity

- Cross section as function of the energy
  Excitation function $\sigma(E_{cm})$

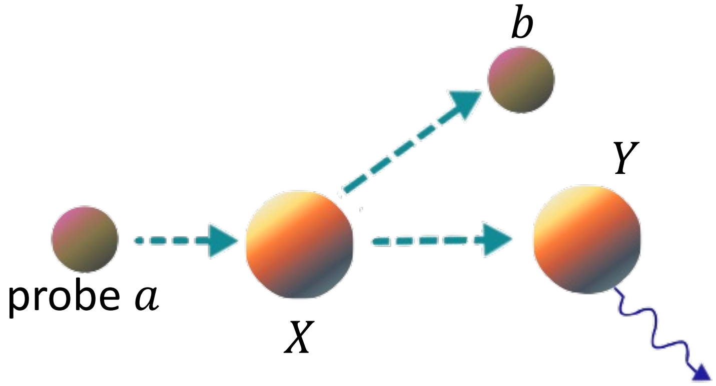
- Cross section for all energies
  smaller than the beam energy:
  stopping the beam in the target
  → total probability for a given channel



Nuclear reactions

- Collision between a beam particle a and a target nucleus X
  In the collision they exchange energy, momentum and possibly mass
  As a result, we obtain a product nucleus Y and
  some outgoing radiation b (particle, γ-ray)

$$a + X \rightarrow b + Y$$



- Alternative notation:

$$X(a,b)Y$$

  Puts the accent on the process (a, b)

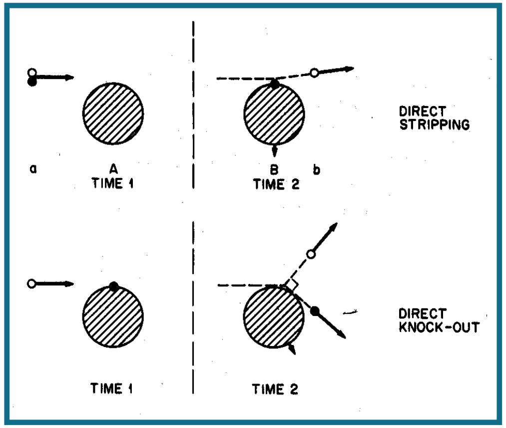
Types of reactions (list not exhaustive)

- Combination of produced particles/radiation: reaction **channel**
- Different channels may be present (**open**) at the same time depending on conservation principles

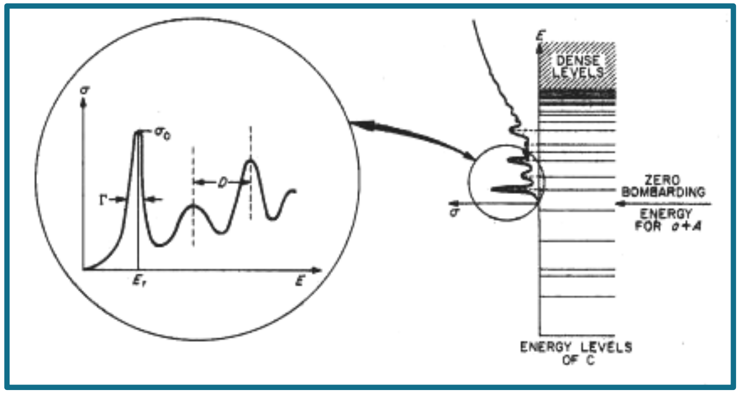
- Elastic scattering: $X(a,a)X$ — $^{12}C(p,p)^{12}C$ $^{208}Pb(n,n)^{208}Pb$
  Always present!
- Inelastic scattering: $X(a,a')X^*$ — $^{12}C(p,p')^{12}C^*$ $^{40}Ca(\alpha,\alpha')^{40}Ca^*$
- Rearrangement reactions: (ex)change of mass
  - Transfer reactions:
    - stripping — $^{12}C(d,p)^{13}C$
    - pick-up — $^{12}C(p,d)^{11}C$
  - Knock-out reactions: — $^{12}C(p,2p)^{11}B$
- Photo-disintegration: $X(\gamma,a)Y$ — $^{16}O(\gamma,\alpha)^{12}C$
- Capture reactions: $X(a,\gamma)Y$ — $^{14}N(\alpha,\gamma)^{18}F$

Time scales

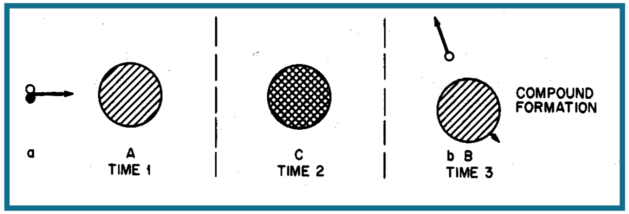
- Direct reactions: transfer, breakup $A(a,c+d)A$
  - Fast, only few nucleons involved
  - Likely to occur at small exit angles (peripheral)
  - Modelled as one-step processes
  - Time scale $\tau \ll 10^{-22}$ s



- Resonance reactions
  - Some nucleons form a resonance that lives for a short time $\tau \approx \hbar/\Gamma$
  - The total kinetic energy matches the energy of a resonance in the compound system



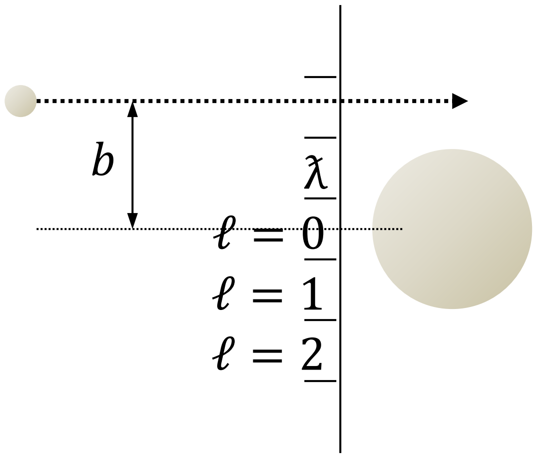
- Compound-nucleus reactions
  - $A+a \rightarrow C^* \rightarrow B+b$
  - Energy is shared among all nucleons
  - Overlap of many resonances, described statistically
  - No memory of entrance channel
  - Products emitted isotropically
  - Time scale $\tau \approx 10^{-22}$ s (time of a nucleon orbital period)



Impact parameter – Angular momentum

- b is the impact parameter
- If $b \approx R_A + R_a$
  peripheral reactions (direct)
- If $b < R_A + R_a$
  compound nucleus / fusion
- Measured as orbital angular momentum:
  $\ell = \frac{1}{\hbar} mvb \rightarrow \boxed{b = \ell ƛ}$
  with $ƛ = \left(\frac{1}{\hbar}\sqrt{2mE_k}\right)^{-1}$ de Broglie wavelength
  (m mass, E_k kinetic energy, in the center-of-mass system)

Impact parameter measured in units of $ƛ$: $b = \ell ƛ$

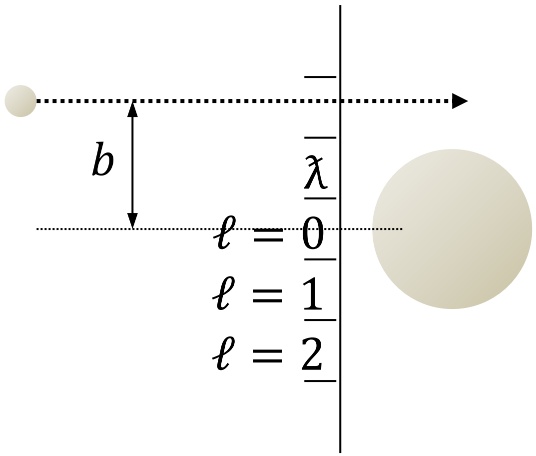


KU LEUVEN

Impact parameter – Angular momentum

- $\ell > (R_A + R_a)/ƛ = \ell_c$ no reaction (possibly Coulomb elastic)
- $\ell \approx \ell_c$ peripheral, direct reactions
- $\ell < \ell_c$ head-on collision, compound nucleus reaction / fusion
- $ƛ$ decreases and ℓ_c increases with increasing energy: more units of ℓ can lead to reaction

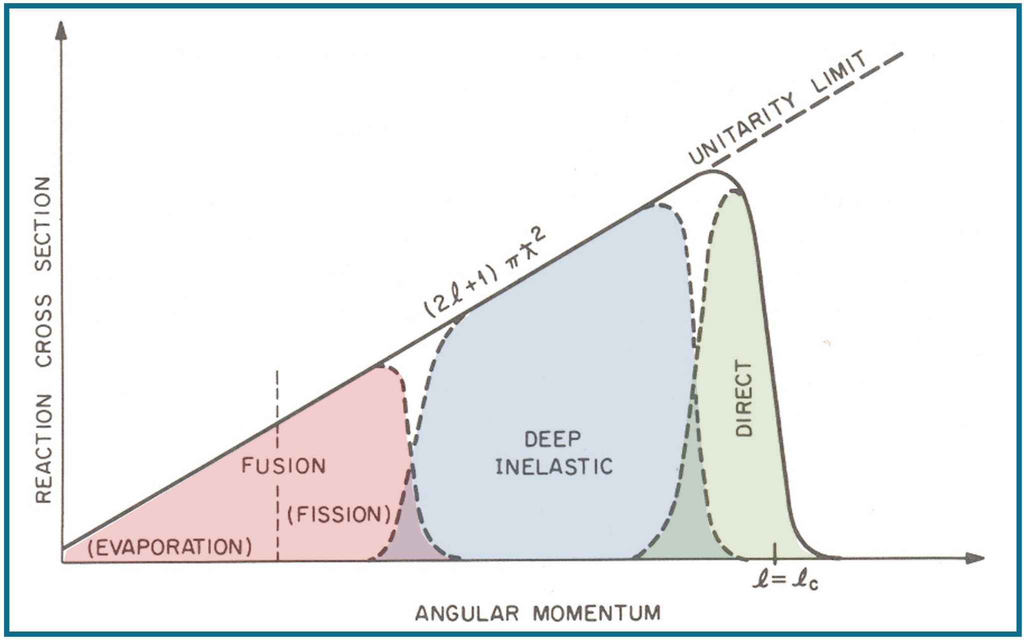
Impact parameter measured in units of $ƛ$: $b = \ell ƛ$



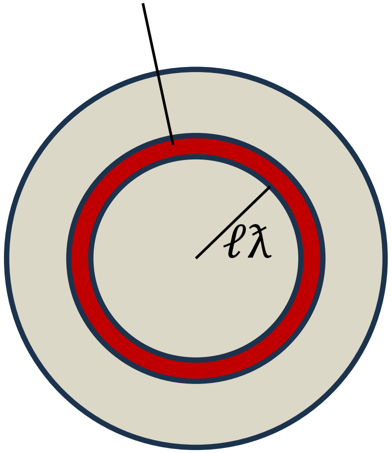
Impact parameter and reaction probability

First rough estimate of the cross section:

Probability of reaction ≈ area of the target

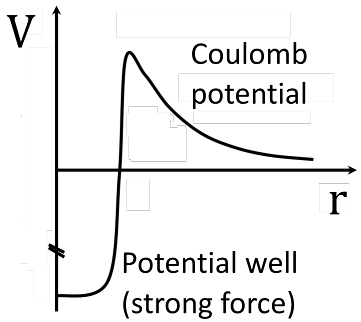


$$\pi[(\ell+1)\lambda\!\!\!^{-}]^2 - \pi[\ell\lambda\!\!\!^{-}]^2 = (2\ell+1)\pi\lambda\!\!\!^{-2}$$

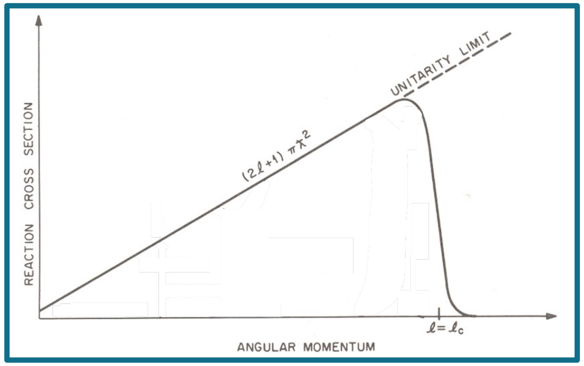


Impact parameter and reaction probability

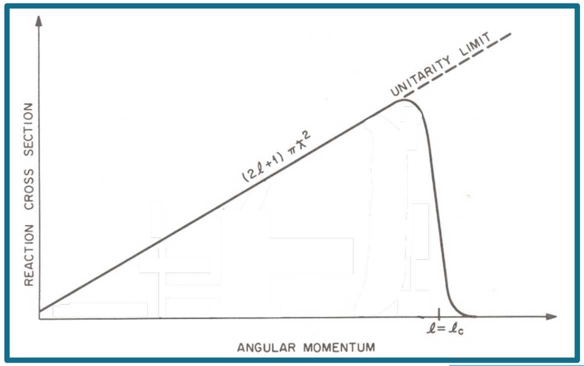
- Charged particles:
  - Sufficient energy to overcome the Coulomb repulsion



- Heavy ions:
  - direct processes negligible
  - deep inelastic increasing with collision energy

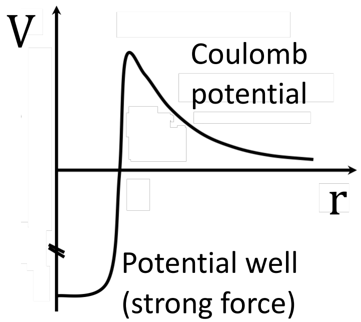


- Light ions (p,n,d,t,^{3}He,α):
  - deep inelastic negligible
  - importance of resonances (at collision energies around the barrier)
  - high energy: fusion → spallation/fragmentation

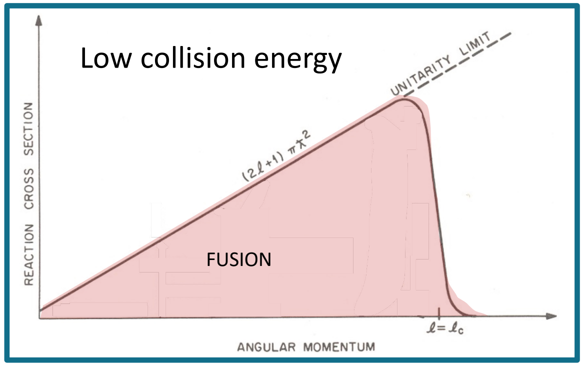


Impact parameter and reaction probability

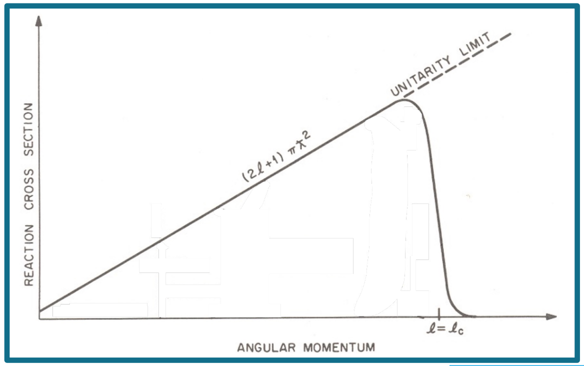
- Charged particles:
  - Sufficient energy to overcome the Coulomb repulsion



- Heavy ions:
  - direct processes negligible
  - deep inelastic increasing with collision energy

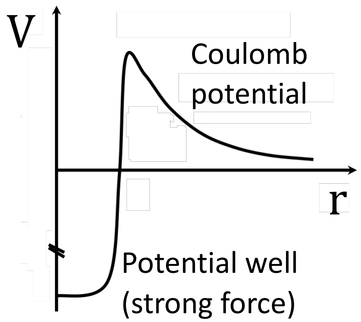


- Light ions (p,n,d,t,^{3}He,α):
  - deep inelastic negligible
  - importance of resonances (at collision energies around the barrier)
  - high energy: fusion → spallation/fragmentation

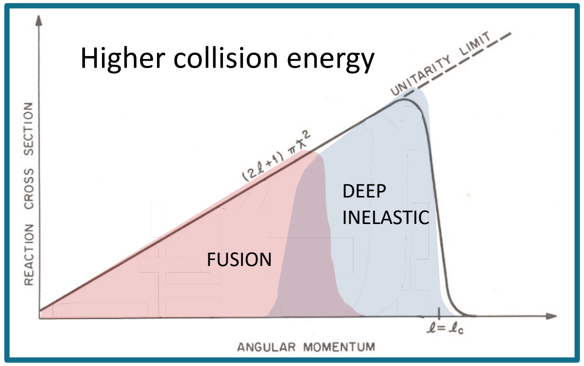


Impact parameter and reaction probability

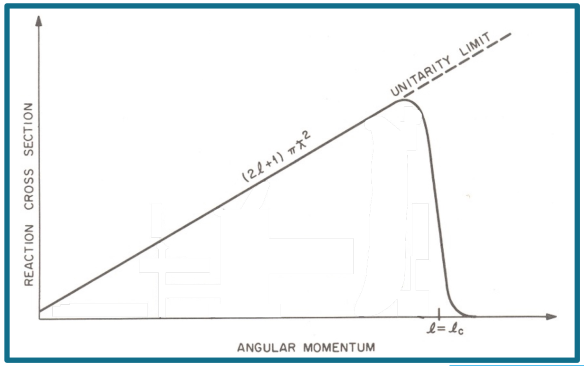
- Charged particles:
  - Sufficient energy to overcome the Coulomb repulsion



- Heavy ions:
  - direct processes negligible
  - deep inelastic increasing with collision energy

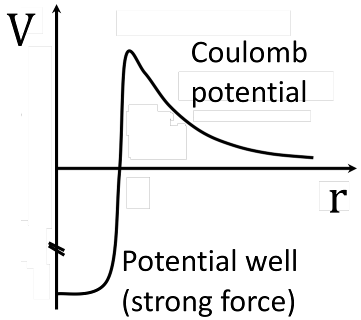


- Light ions (p,n,d,t,^{3}He,α):
  - deep inelastic negligible
  - importance of resonances (at collision energies around the barrier)
  - high energy: fusion → spallation/fragmentation

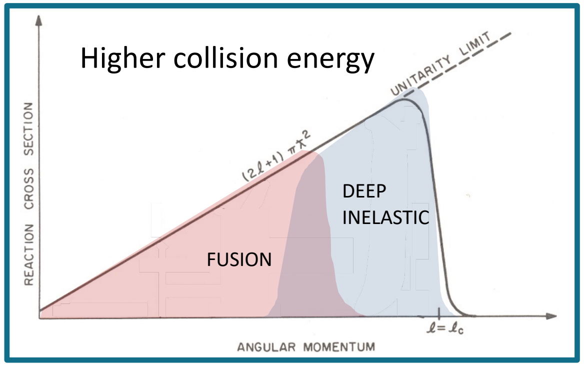


Impact parameter and reaction probability

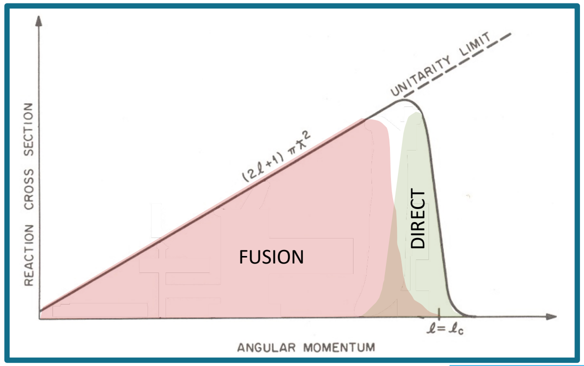
- Charged particles:
  - Sufficient energy to overcome the Coulomb repulsion



- Heavy ions:
  - direct processes negligible
  - deep inelastic increasing with collision energy

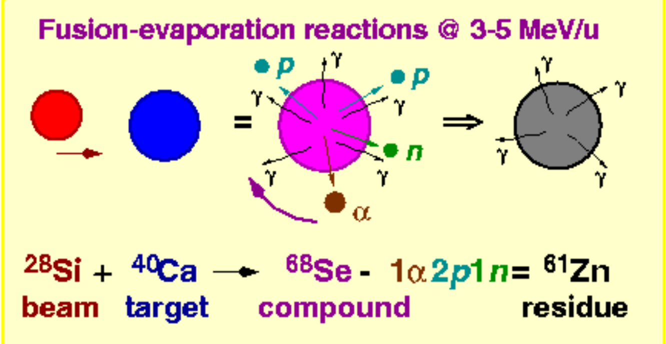


- Light ions (p,n,d,t,^{3}He,α):
  - deep inelastic negligible
  - importance of resonances (at collision energies around the barrier)
  - high energy: fusion → spallation/fragmentation

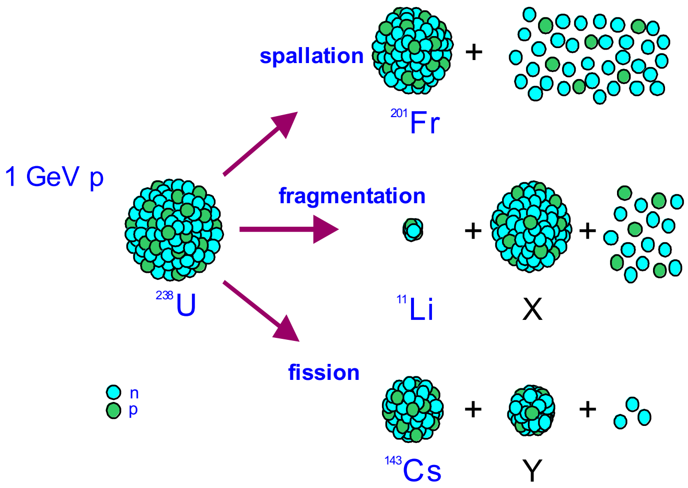


Reaction processes

- Fusion: two-step process
  reaction + "decay" (fission, evaporation)



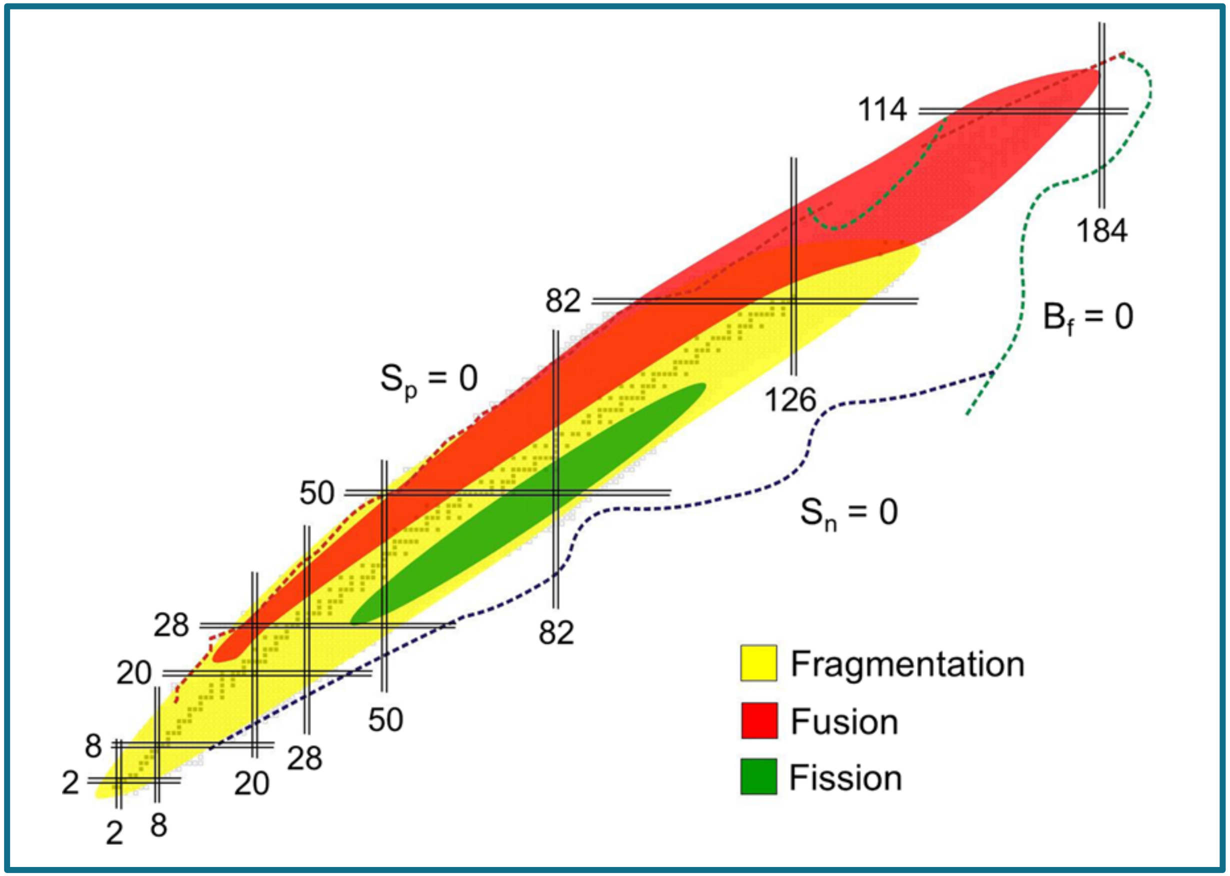
- Higher energies:
  - fragmentation (few 100 MeV/nucleon)
  - spallation (≈GeV/nucleon)



Cross sections do not depend upon the kinematics!

However, in inverse kinematics (in-flight separation)
the production is more directly related to the cross section

Production of unstable nuclei



G. G. Adamian et al., Eur. Phys. J. A 56:47 (2020)

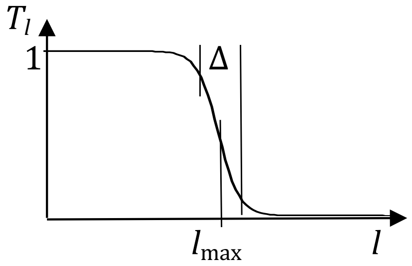
Modelling fusion cross sections

1. **Compound nucleus formation**
   For each angular momentum:

$$\sigma_l = \pi \lambda\!\!\!^{-2} (2l + 1) T_l$$

with

$$T_l = \left[1 + \exp\left(\frac{l - l_{max}}{\Delta} \right) \right]^{-1}$$



2. **Evaporation probabilities / Fission: statistical equilibrium**
   Calculated as the cross section of the inverse reaction
   Ingredients:
   - *E** of compound nucleus and of residue
   - separation energy (for particles)
   - level density: parametrised

Fission occurs if $l_{max} \geq l_{crit}$ at which the fission barrier becomes smaller than the neutron separation energy

Calculating fusion cross sections

Codes

- PACE (1980)
  Monte-Carlo
- CASCADE (1977)
  Analytic
- HIVAP (1981)
  Analytic
- CNABLA (1999)
  Monte-Carlo
- GEMINI++ (2008)
  Monte-Carlo

Nuclear Inst. and Methods in Physics Research B 416 (2018) 41–49

Contents lists available at ScienceDirect

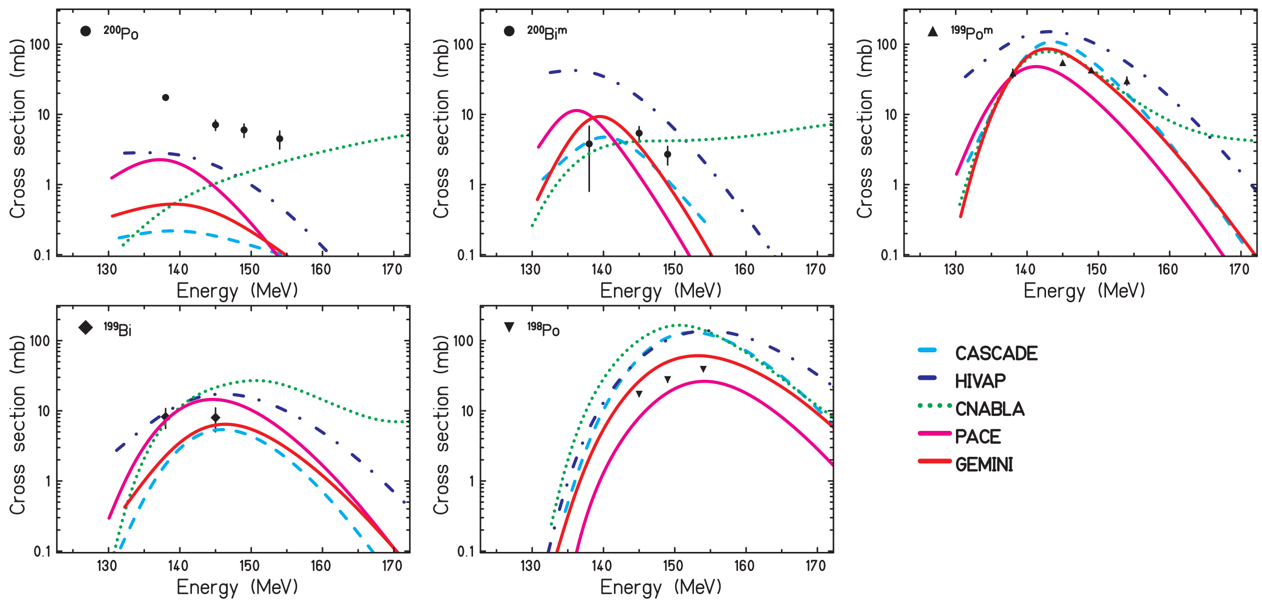
Nuclear Inst. and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

ELSEVIER

Evaluation of fusion-evaporation cross-section calculations

B. Blank[a,b,*], G. Canchel[a], F. Seis[a,1], P. Delahaye[c]



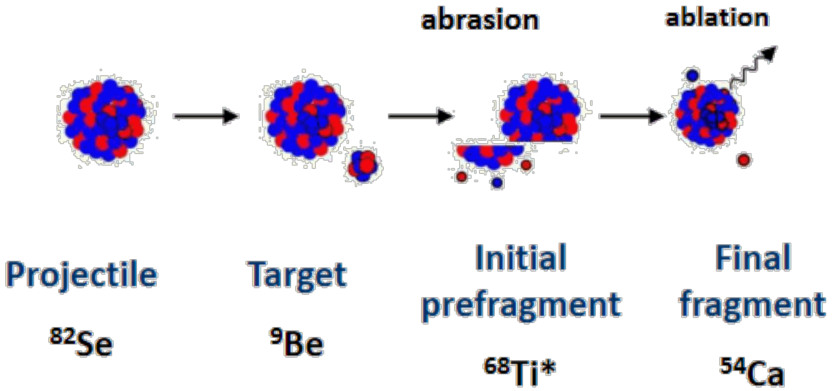
In general:
overestimation of experimental cross sections

KU LEUVEN

Cross sections at relativistic energies

ABRABLA07

J.-J. Gaimard, K.-H. Schmidt, Nucl. Phys. A 531 (1991) 709



1. **Abrasion**
   - (only) nucleons with overlapping trajectories collide
   - Excitation energy from holes in the level scheme

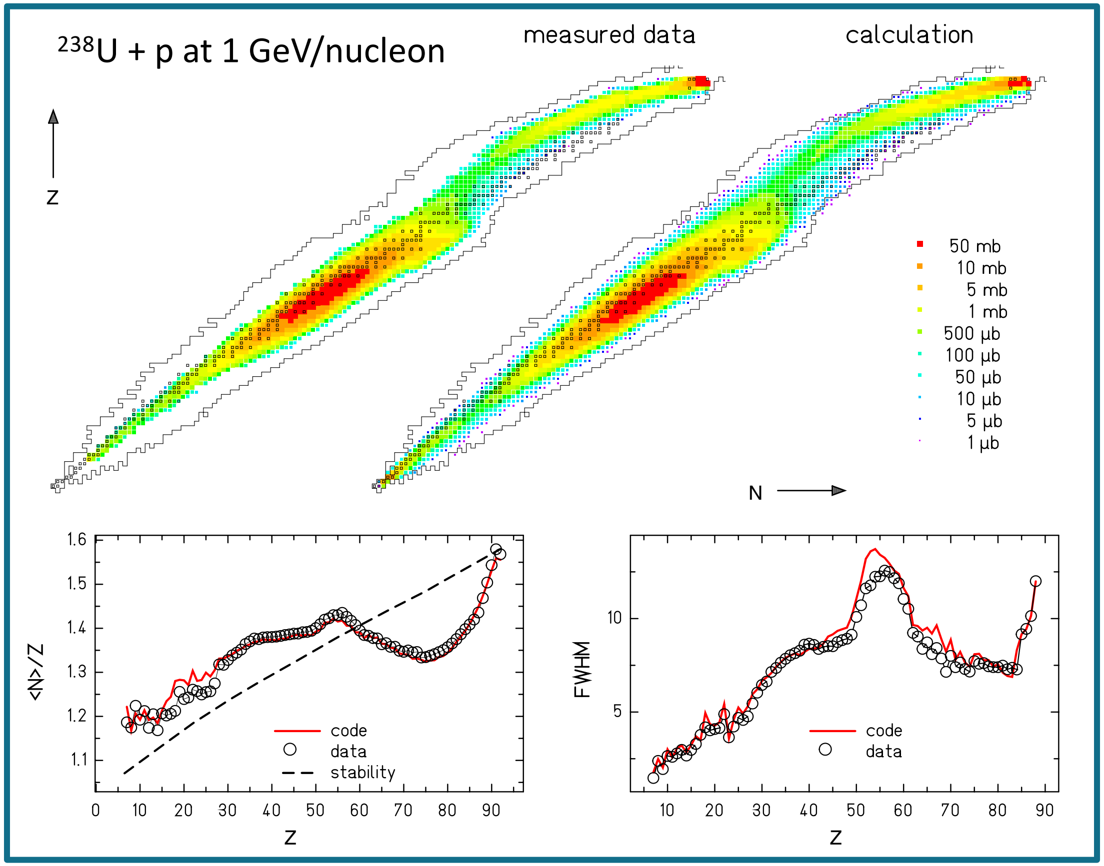
2. **(Possibly) breakup of the prefragment**
   For very high excitation energies
   N/Z ratio is conserved

3. **De-excitation (ablation)**
   Competition between evaporation (of n,p,d,t,α,γ) and fission

Cross sections at relativistic energies

ABRABLA07

J.-J. Gaimard, K.-H. Schmidt, Nucl. Phys. A 531 (1991) 709



S. Lukić et al., Nuclear Instruments and Methods in Physics Research A 565 (2006) 784

Cross sections at relativistic energies

Nuclear Inst. and Methods in Physics Research B 416 (2018) 41–49

ELSEVIER

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Evaluation of fusion-evaporation cross-section calculations

B. Blank[a,b,*], G. Canchel[a], F. Seis[a,1], P. Delahaye[c]

Other Codes

- EPAX (1990)
  Phenomenological formula for fragmentation yields
  Parameters fitted on experimental values

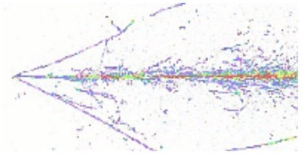
- SPACS (2014)
  Semi-empirical parameterization of spallation yields

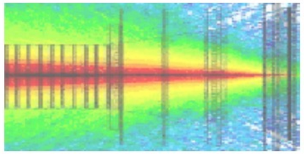
- GRAZING (1995)
  Model (and code) for deep-inelastic reactions

KU LEUVEN

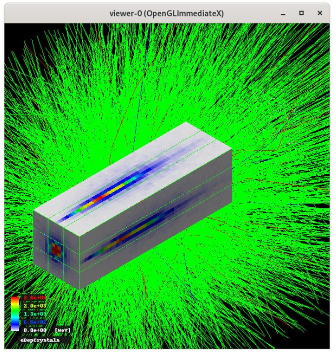
Comprehensive simulation codes

- **FLUKA** http://www.fluka.org
  - "Fully integrated particle physics Monte-Carlo simulation package"
  - Calculations of particle transport and interactions with matter

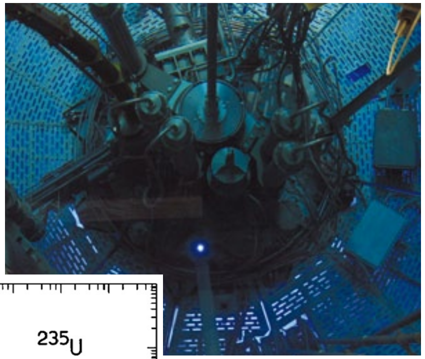




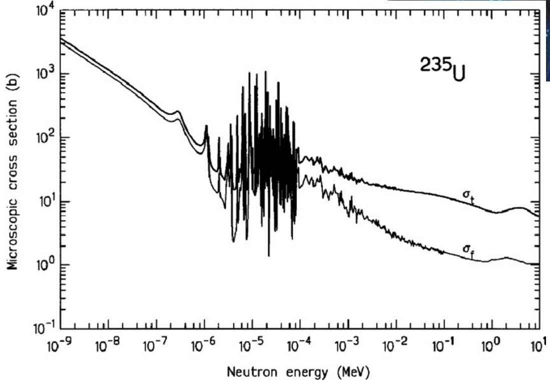
- **GEANT4** https://geant4.web.cern.ch
  - "Simulation of the passage of particles thorugh matter"



Current production of radioisotopes



- **Neutron capture** on stable targets
  Production of slightly neutron-rich isotopes
  Mainly at reactor facilities
  Resonances are important!



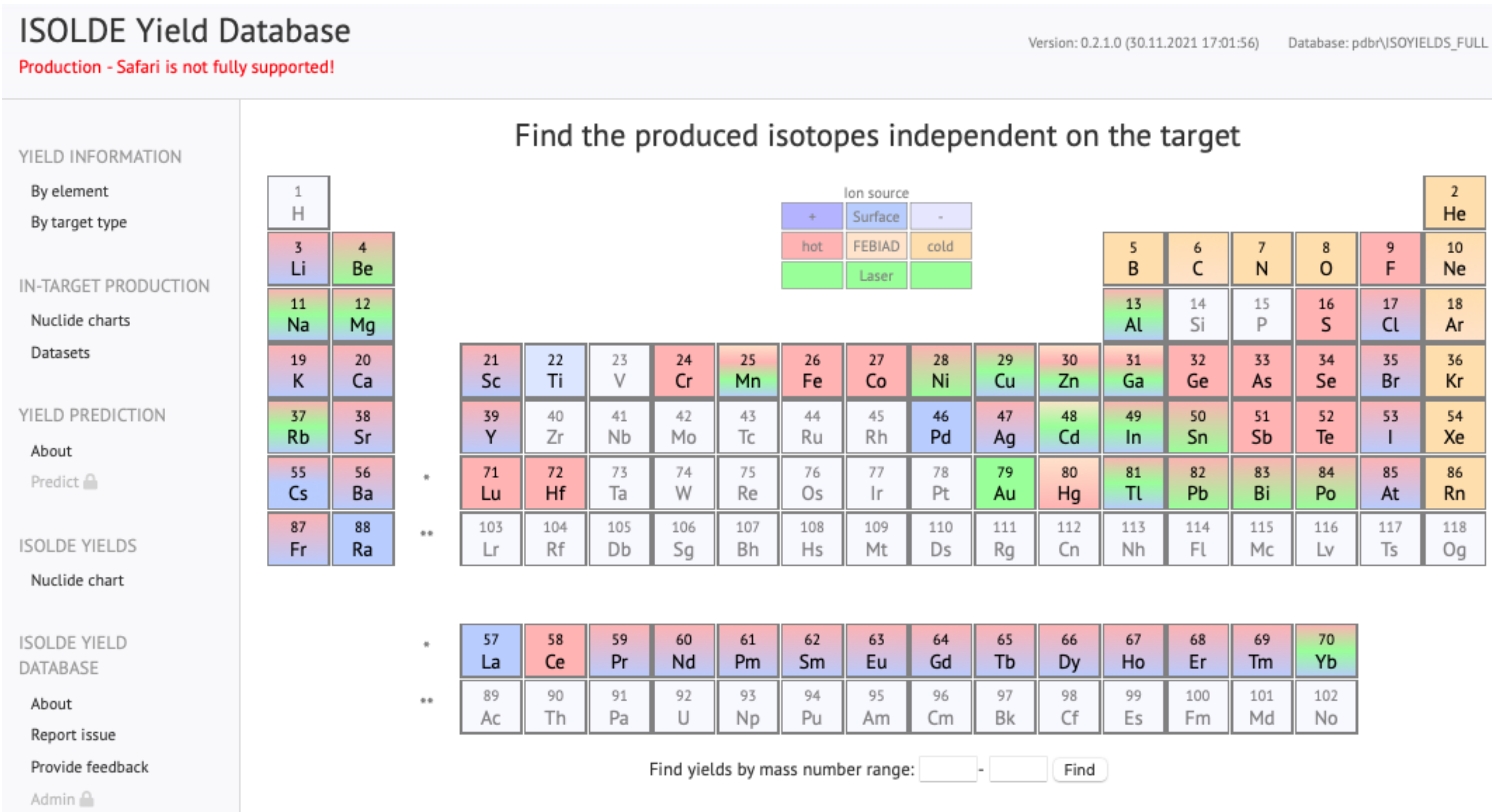
- **Fission**
  Thermal neutrons on ^{235}U
  Fast neutrons on ^{238}U

- **Proton-induced reactions** (p,xn), (p,α)
  For positron emitters
  Dedicated cyclotrons

- **Light ion-induced reactions** (d,xn), (d,α), (α,xn)
  Dedicated cyclotrons

Yields at Radioactive Ion Beam facilities

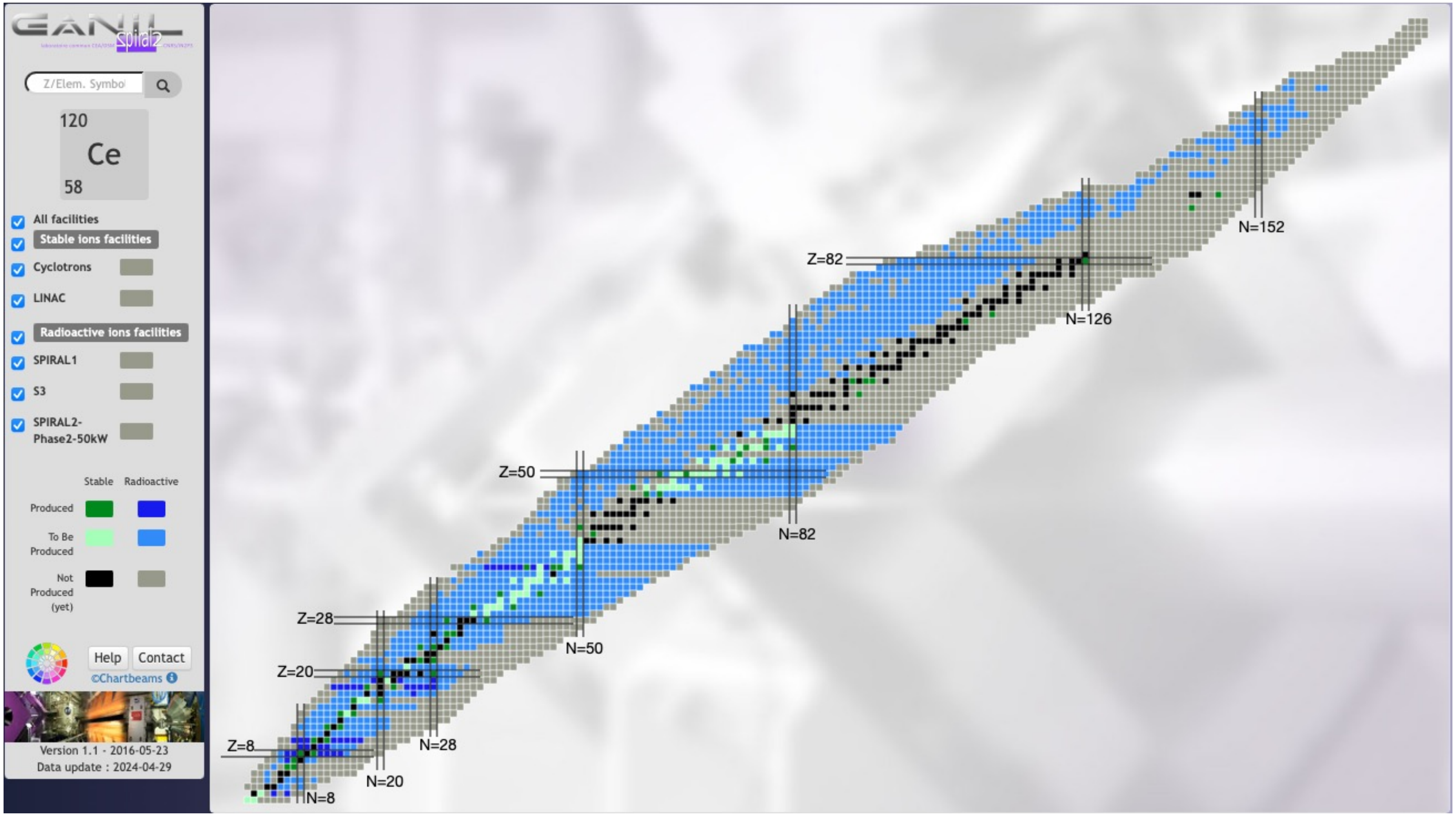
ISOLDE https://isoyields2.web.cern.ch



KU LEUVEN

Yields at Radioactive Ion Beam facilities

GANIL/SPIRAL(2) https://u.ganil-spiral2.eu/chartbeams/



KU LEUVEN

Summary/conclusions

- Cross sections are <u>very</u> difficult to calculate
  → use semi-classical models
  → use phenomenological approaches
- The most useful reaction process is fusion
  Two-step process
  Modelling evaporation is challenging!
  Several codes available
- New radioisotopes: challenging research!
  Information from radioactive ion beam facilities

The cross section is only the first step

…thank you for your attention!

Enjoy Leuven and Belgium!

KU LEUVEN